Classifying spaces for families of subgroups

Let G — discrete group

$\mathcal{F}$ — family of subgroups of G

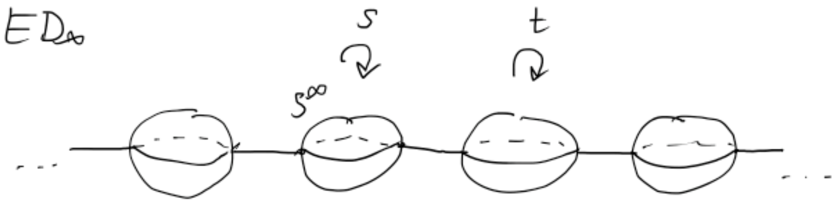
(e.g. $TR = \{\{e\}\}$, $FIN = \{H \subset G \mid H \text{ finite}\}$, ...)

<u>Def</u>: A model for $\boxed{E_{\mathcal{F}} G}$ is a G-CW-complex X s.t. for $H \subset G$:

$$X^H \text{ is } \begin{cases} \simeq * , & \text{if } H \in \mathcal{F} \\ = \emptyset , & \text{if } H \notin \mathcal{F} \end{cases}.$$

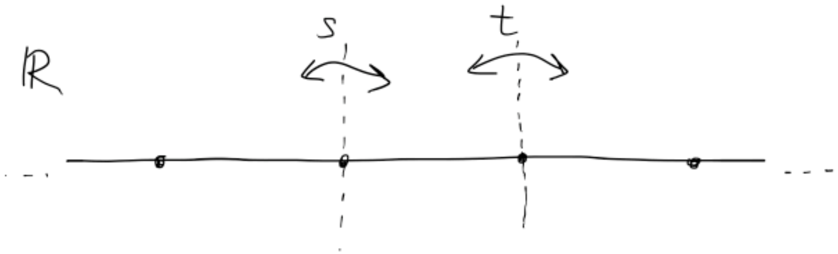
<u>Ex</u>: $G = D_\infty = \mathbb{Z}/2 * \mathbb{Z}/2 = \langle s, t \mid s^2 = t^2 = e \rangle$.

$$E_{TR} D_\infty = E D_\infty$$



$$\downarrow$$

$$E_{FIN} D_\infty = \mathbb{R}$$



Kevin Li
University of Southampton

Nansen Petrosyan
Ian Leary

① Bounded cohomology

$$\boxed{H^n_b(E_{\mathcal{F}} G)} \longrightarrow H^n(E_{\mathcal{F}} G)$$

characterizes:

- co-amenability
- relative hyperbolicity.

② LS-category, Farber's TC, ...

$$\boxed{\mathrm{cat}_{\mathcal{F}}(G)} := \inf \left\{ n \in \mathbb{N} \;\middle|\; \begin{tikzcd} EG \arrow[r] \arrow[dr, dashed] & E_{\mathcal{F}} G \arrow[d, phantom, "\cup"] \arrow[dl, phantom, "\simeq"] \\ & n\text{-skel}(E_{\mathcal{F}} G) \end{tikzcd} \right\}$$

$$\vee\!|$$

$$\sup \{ n \in \mathbb{N} \mid H^n(E_{\mathcal{F}} G) \longrightarrow H^n(EG) \text{ is non-trivial} \}.$$